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Abstract

We consider the dynamics of tensor and scalar gravitational fields in the relativistic theory of gravitation with the Minkowskian vacuum metric and generalize the formulation to the lifted degeneration of masses. The potential of scalar field is uniquely determined in the presence of cosmological term. We find cosmological inflationary solutions and analyze conditions providing the transition to the regime of hot expanding Universe.

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1 Introduction

Recently the astronomical observations on Supernovas with high red shifts [1, 2] shown that the most probable value for the fraction of cosmological term in the density of energy differs from zero even in the case of flat Universe (see Fig. 1 from [1]). Such the measurements result in the average values of parameters determining the fractions of energy densities for the matter and cosmological contribution in ratios to the critical density of energy for the flat Universe, so that $\Omega_M = 0.28^{+0.09+0.05}_{-0.08-0.04}$ and $\Omega_\Lambda \approx 0.72$, respectively, with the same error bars.

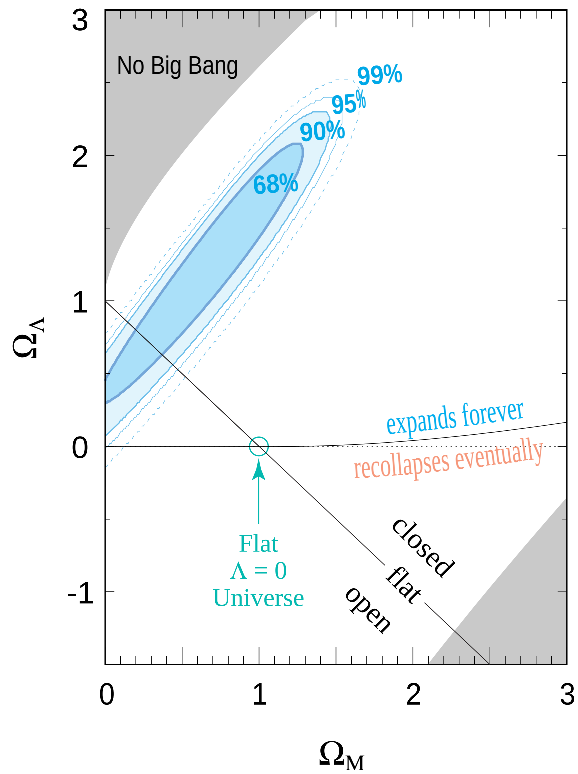

Figure 1: Results of [1] represented in the plane of Ω_M and Ω_Λ parameters. The flat Universe corresponds to the line $\Omega_\Lambda + \Omega_M = 1$.

For the nonrelativistic motion of matter the deceleration parameter of Universe expansion is determined by the expression

$$q = \frac{\Omega_M}{2} - \Omega_\Lambda,$$

which leads to the estimate [1, 2, 3]

$$q = -0.33 \pm 0.17. \tag{1}$$

Thus, the experimental data point to the actuality and necessity of comprehensive analysis on the evolution of flat Universe in the presence of cosmological term.

The cosmological constant in the flat Universe is a necessary ingredient in the relativistic theory of gravitation (RTG) [4]. The construction of RTG by A.A.Logunov was based on the fundamental physical principle of vacuum stability, so that, according to RTG, the vacuum is the flat Minkowskian space-time. In this way the metric tensor of Riemannian space-time is considered as the Faraday–Maxwell field over the flat vacuum. The field equations of motion have to satisfy the following principles:

1. The vacuum stability: the gravitation field equations are identically true under the absence of matter fields, so that the metric tensor of Riemannian space-time is equal to the Minkowskian metric.
2. The conservation law: the divergence of energy-momentum tensor for the gravitational field under the covariant derivative composed by the connection consistent with the Minkowskian metric is identically equal to zero.
3. The weak equivalence principle (the geometrization): the interaction of gravitational field with the matter is built under the Riemannian metric.

The theory of gravitational field constructed in [4] leads to the Lagrangian, which coincides with the Einstein–Hilbert Lagrangian in the theory of general relativity, if only the cosmological constant is equal to zero. A nonzero cosmological term implies the introduction of additional contribution, which is **uniquely** determined in RTG and related with the mass of graviton. This mass can be introduced in the presence of background, vacuum metric, the Minkowskian one in the case under study. In [4] the theory with the degenerated masses of tensor graviton (spin 2) and scalar gravitational field (spin 0) was constructed. The additional fields of spins 1 and 0 are eliminated from RTG due to the consequence from the conservation law (the second principle), which makes the divergence of Riemannian metric under the covariant Minkowskian derivative be equal to zero. Thus, the RTG is a bi-metric theory reducible to a scalar-tensor variant for the extension[1] of standard Lagrangian for the gravitational field in the general relativity, but the ambiguity in the choice of functional parameters in the theory is completely cancelled due to the clear physical restrictions given above.

The motivation of RTG in the light of modern view to the problem is not restricted by the principle to construct the theory for the Faraday–Maxwell field of gravitation. Indeed, the geometrization principle for the interaction of gravity with the matter leads to the non-renormalizability of quantum theory as in the general theory of relativity. However, we can suggest that we deal with the classical low-energy field theory under virtualities less than the Planck scale, while a full renormalizable theory is formulated in the flat space-time[2], and the dimensional Planck scale appears under the breaking of conformal invariance in the renormalization group and the spontaneous breaking of higher symmetry in the full theory [6]. In this way the stable vacuum in the form of Minkowskian metric can enter the effective low-energy Lagrangian for the gravitational field, that takes place in RTG in the case of nonzero cosmological constant. On the other hand, if we part with the principle of renormalizability, then we can suppose that a full theory is not local, i.e. it deals with some extended objects, that necessarily leads to an extension of full high-energy space-time dimension [7]. These additional dimensions have to be compactified at the energies below the Planck scale, so that the vacuum is determined by a configuration, which can enter the effective classic theory of gravity. In the case of flat four-dimensional vacuum, the Lagrangian can depend on the Minkowskian metric, that again points to the actuality of consideration with the theory-construction principles accepted in RTG.

[1]Some examples of bi-metric theories of gravity and scalar-tensor variants are presented in [5].
[2]In the curved space-time the local quantum field theory is certainly nonrenormalizable.

In [4] the gravitational field equations are derived in the case of degenerate masses of tensor and scalar fields. In the study of Universe evolution in that version of RTG, authors found stringent restrictions on the graviton mass, that followed from the age of Universe. The cosmological solution in the presence of matter was found pulsating with the non-inflationary expansion, which has lots of problems [8]. Moreover, the deceleration parameter occurred greater than $\frac{1}{2}$, that contradicts the mentioned astronomical observations. In general, the rather strict suggestion on the degeneration of masses can be lifted out. In the present paper we modify the RTG by studying non-degenerated tensor and scalar gravitational fields in the presence of nonzero cosmological term. The exploration of RTG principles leads to an unique determination of potential for the scalar field, which equation of motion can be exactly integrated out. In this way the tensor graviton with spin 2 occurs massless, and the dimensional parameter giving the cosmological constant determines the mass term of scalar field. The offered modification of RTG results in significant cosmological implications. Namely, the evolution of Universe at the initial stage has the inflationary solution (see a comprehensive description of inflationary scenario in [8]). The value of cosmological constant falls under the inflation, so that at late times under the transition to flat expanding Universe the deceleration parameter agrees with the recent astronomical observations. We show at which conditions the equations of motion in the cosmology lead to the regime of hot expanding Universe.

In Section 2 we construct the dynamics of RTG. Then we find the solutions for the evolution of Universe in the presence of cosmological constant in RTG with the dust or ultra-relativistic matter in Section 3. The obtained results are summarized and discussed in Conclusion.

2 Dynamics of tensor and scalar gravitational fields in RTG

According to RTG the density of Riemannian metric $\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$ is expressed in terms of sum of densities for the Minkowskian metric and gravitational Faraday–Maxwell field

$$\begin{aligned} \mathfrak{g}^{\mu\nu} &= \sqrt{-\gamma}\,(\gamma^{\mu\nu} + \Phi^{\mu\nu}), \\ D_\mu\, \Phi^{\mu\nu} &= 0, \end{aligned} \tag{2}$$

where D_μ is the covariant derivative with the Christoffel symbols determined by the Minkowskian metric, so that they globally become zero in the Galilean (Cartesian) coordinates in the whole space-time, while the zero divergence actually is the implication of conservation law for the energy-momentum tensor, that will be shown later. The gravitational field can be expanded to the traceless tensor part and the scalar contribution, so that

$$\begin{aligned} \Phi^{\mu\nu} &= \left[\Phi^{\mu\nu} - \tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\,(\gamma_{\alpha\beta}\Phi^{\alpha\beta})\right)\right] + \tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\right)\omega' \\ &= u^{\mu\nu} + w'^{\mu\nu}, \\ D_\mu\, u^{\mu\nu} &= 0, \quad \gamma_{\mu\nu}\, u^{\mu\nu} = 0, \\ \mathfrak{u}^{\mu\nu} &= \sqrt{-\gamma}\, u^{\mu\nu} = \sqrt{-\gamma}\,\left[\Phi^{\mu\nu} - \tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\,(\gamma_{\alpha\beta}\Phi^{\alpha\beta})\right)\right], \\ \mathfrak{w}'^{\mu\nu} &= \sqrt{-\gamma}\, w'^{\mu\nu} = \sqrt{-\gamma}\,\tfrac{1}{3}\left(\gamma^{\mu\nu} - \tfrac{D^\mu D^\nu}{D^2}\right)\omega'. \end{aligned} \tag{3}$$

We introduce an independent density caused by the scalar field in the form

$$\mathfrak{w}^{\mu\nu} = \mathfrak{w}'^{\mu\nu} + \sqrt{-\gamma}\,\gamma^{\mu\nu} = \sqrt{-\gamma}\,\frac{1}{3}\left(\gamma^{\mu\nu} - \frac{D^\mu D^\nu}{D^2}\right)\omega, \tag{4}$$

where $\omega = 4 + \omega'$, and at zero ω' we have got the flat metric. It is significant that the contraction of Riemannian and Minkowskian metric tensors depends on the scalar component of gravitational field, only, so that

$$\gamma_{\mu\nu}\mathfrak{g}^{\mu\nu} = \gamma_{\mu\nu}\mathfrak{w}^{\mu\nu},$$

where, in accordance with our definitions, we put

$$\mathfrak{g}^{\mu\nu} = \mathfrak{u}^{\mu\nu} + \mathfrak{w}^{\mu\nu}.$$

The density of Lagrangian for the gravitational field in the modified RTG with the cosmological constant is expressed in the form[3]

$$\mathfrak{L} = -\frac{1}{4}\,\mathfrak{R} + \lambda_1\sqrt{-g} + \lambda_2\sqrt{-g}\,\gamma_{\mu\nu}w^{\mu\nu} + \sqrt{-\gamma}\,V(\omega), \tag{5}$$

where $V(\omega)$ is an arbitrary function of scalar field.

Then the tensor field equations of motion have got the form

$$\frac{\delta\mathfrak{L}}{\delta\mathfrak{u}^{\mu\nu}} = -\frac{1}{4}\,R_{\mu\nu} + \frac{1}{2}\,\lambda_1\,g_{\mu\nu} + \frac{1}{2}\,\lambda_2\,g_{\mu\nu}\,\gamma_{\alpha\beta}w^{\alpha\beta} = 0, \tag{6}$$

According to the RTG principle of vacuum stability, in the limit of $u^{\mu\nu} \to 0$, $w^{\mu\nu} \to \gamma^{\mu\nu}$ the purely gravitational equations of motion with no matter have to be identically satisfied, so that we get

$$\lambda_1 = -4\lambda_2. \tag{7}$$

The scalar field equation of motion, taking into account the above tensor field equations, gives the expression

$$\begin{aligned}\frac{\delta\mathfrak{L}}{\delta\omega} &= \sqrt{-\gamma}\,\frac{1}{3}\left(\gamma^{\mu\nu} - \frac{D^\mu D^\nu}{D^2}\right)\left(-\frac{1}{4}\,R_{\mu\nu} + \frac{1}{2}\,\lambda_1\,g_{\mu\nu} + \frac{1}{2}\,\lambda_2\,g_{\mu\nu}\,\gamma_{\alpha\beta}w^{\alpha\beta}\right) + \\ &\quad +\lambda_2\,\sqrt{-g} + \sqrt{-\gamma}\,V'(\omega) = \\ &= \lambda_2\,\sqrt{-g} + \sqrt{-\gamma}\,V'(\omega) = 0,\end{aligned} \tag{8}$$

so that in the limit of flat vacuum we find

$$V'(\omega)\big|_{\omega'=0} = -\lambda_2. \tag{9}$$

After taking into account the field equations, the density of energy-momentum tensor for the gravitational field can be written down in the form

$$\mathfrak{t}_g^{\mu\nu} = -2\frac{\delta\mathfrak{L}}{\delta\gamma_{\mu\nu}} = -\frac{1}{4}\,\mathfrak{J}^{\mu\nu} - 2\,\lambda_2\,\sqrt{-g}\,w^{\mu\nu} - \sqrt{-\gamma}\,\gamma^{\mu\nu}\,V(\omega), \tag{10}$$

where the density of current [4] is equal to

$$\mathfrak{J}^{\mu\nu} = D_\alpha D_\beta(\gamma^{\alpha\mu}\mathfrak{g}^{\beta\nu} + \gamma^{\alpha\nu}\mathfrak{g}^{\beta\mu} - \gamma^{\alpha\beta}\mathfrak{g}^{\mu\nu} - \gamma^{\mu\nu}\mathfrak{g}^{\alpha\beta}).$$

[3] We accept the units, in which $4\pi G = 1/m_{\rm PL}^2 = 1$, G is the gravitational constant, $m_{\rm PL}$ is the Planck mass.

In the absence of gravitational field, the vacuum solution have to lead to zero of introduced energy-momentum tensor, hence, we find

$$V(\omega)|_{\omega'=0} = -2\lambda_2. \tag{11}$$

The scalar field equation of motion (8) implicates a stringent restriction to the potential

$$\lambda_2 \sqrt{-g} = -\sqrt{-\gamma}\, V'(\omega), \tag{12}$$

which use provides the following form of energy-momentum tensor:

$$\mathfrak{t}_g^{\mu\nu} \;=\; -\frac{1}{4}\,\mathfrak{J}^{\mu\nu} + \sqrt{-\gamma}\,\gamma^{\mu\nu}\,[2V'(\omega) - V(\omega)] + 2\,V'(\omega)\,\mathfrak{w}'^{\mu\nu}, \tag{13}$$

Then the covariant conservation law is valid, if we put

$$\left.\begin{array}{rcl} D_\mu\,\mathfrak{g}^{\mu\nu} &=& 0,\\ 2V'(\omega) &=& V(\omega), \end{array}\right\} \quad\Longrightarrow\quad D_\mu\,\mathfrak{t}_g^{\mu\nu} = 0. \tag{14}$$

Therefore, in the modified RTG the potential of scalar field is uniquely determined, and the motion equation can be exactly integrated out, so that

$$\begin{array}{rcl} V(\omega') &=& -2\lambda_2\,\exp\left[\tfrac{1}{2}\omega'\right],\\ \omega' &=& \ln\left[\dfrac{g}{\gamma}\right]. \end{array} \tag{15}$$

After the substitution, the transversity of tensor density $\mathfrak{t}_g^{\mu\nu}$ becomes explicit

$$\frac{1}{4}\,D^2(\mathfrak{u}^{\mu\nu} + \mathfrak{w}'^{\mu\nu}) - 2\lambda_2\,\exp\left[\frac{1}{2}\omega'\right]\,\mathfrak{w}'^{\mu\nu} = \mathfrak{t}_g^{\mu\nu}, \tag{16}$$

whereas for the trace we obtain the expression

$$D^2\omega' - 8\lambda_2\,\omega'\,\exp\left[\frac{1}{2}\omega'\right] = \frac{4}{\sqrt{-\gamma}}\,\mathfrak{t}_g^{\mu\nu}\,\gamma_{\mu\nu}. \tag{17}$$

At small perturbations of gravitational field, the dimensional parameter determining the cosmological constant is related with nonzero mass of scalar field, while the tensor graviton field is massless, indeed,

$$\lambda_2 = -\frac{m_\omega^2}{8}, \quad m_u^2 = 0.$$

Then the density of Lagrangian for the gravitational field can be represented in the form

$$\begin{array}{rcl} \mathfrak{L} &=& -\dfrac{1}{4}\,\mathfrak{R} + \sqrt{-\gamma}\,[4V'(\omega) - V'(\omega)\,\omega + V(\omega)] =\\ &=& -\dfrac{1}{4}\,\mathfrak{R} + \dfrac{m_\omega^2}{8}\sqrt{-\gamma}\,(2-\omega')\exp\left[\dfrac{1}{2}\omega'\right]. \end{array} \tag{18}$$

The effective potential of scalar field in the flat Minkowskian space-time is equal to

$$V_{\rm eff} = -\frac{m_\omega^2}{8}\sqrt{-\gamma}\,(2-\omega')\exp\left[\frac{1}{2}\omega'\right] = \frac{m_\omega^2}{8}\,\sqrt{-\gamma}\,U(\omega'), \tag{19}$$

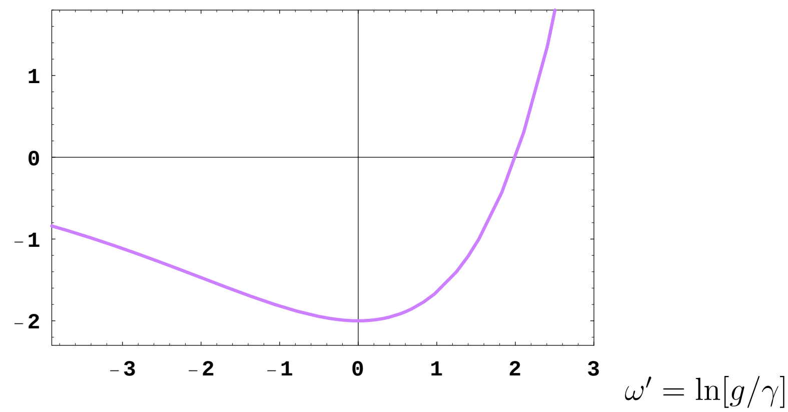

Figure 2: The dependence of scalar field potential in dimensionless units as follows from (19).

where the dimensionless function $U(\omega')$ has got the only minimum corresponding to the flat vacuum of Riemannian space-time (see Fig. 2).

In terms of Riemannian metric, the potential can be transformed to the form

$$V_{\text{eff}} = -\frac{m_\omega^2}{8}\sqrt{-g}\,\left(2 - \ln\left[\frac{g}{\gamma}\right]\right), \tag{20}$$

where the vacuum Minkowskian metric enters explicitly.

It is evident that the geometrization principle for the interaction of gravitation with the matter provides the validity of above conclusions on the form of gravitational Lagrangian. The motion equations are written down as

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R - \frac{m_\omega^2}{4}\ln\left[\frac{g}{\gamma}\right]\,g_{\mu\nu} = 2\,T^M_{\mu\nu}, \tag{21}$$

where the energy-momentum tensor of matter is defined under its Lagrangian density $\mathfrak{L}^M$ by the expression

$$T^M_{\mu\nu} = 2\,\frac{\delta\mathfrak{L}^M}{\sqrt{-g}\delta g^{\mu\nu}}.$$

Thus, we have completely determined the dynamics of gravitational field in RTG with the cosmological constant.

3 Cosmological solutions

In this section we analyze the scenario for the Universe evolution in RTG. From the very beginning it is clear that in RTG due to the principle of vacuum stability the empty flat space-time is the solution of motion equations, and this fact will be illustrated in this section, so that the expansion of Universe containing a matter takes place under an introduction of instability parameter. Therefore, here we will consider the Lagrangian density with the negative square of mass parameter introduced above. Thus, the starting point of our analysis is the gravitation Lagrangian density written down in the form

$$\mathfrak{L} = -\frac{1}{4}\mathfrak{R} - \frac{\mu_\omega^2}{8}\sqrt{-g}\left(2 - \ln\left[\frac{g}{\gamma}\right]\right). \tag{22}$$

Consider the Riemannian metric, which gives the following Friedmann–Robertson–Walker interval:

$$ds^2 = dt^2 - a^2(t)\,[dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)]. \tag{23}$$

The cosmological term can be rewritten as the contribution into the energy-momentum tensor

$$T^\Lambda_{\mu\nu} = 2\,\frac{\delta}{\sqrt{-g}\delta g^{\mu\nu}}\left\{-\frac{\mu_\omega^2}{8}\sqrt{-g}\left(2 - \ln\left[\frac{g}{\gamma}\right]\right)\right\} = -\frac{\mu_\omega^2}{8}\ln\left[\frac{g}{\gamma}\right]\,g_{\mu\nu}. \tag{24}$$

The equations of motion (21) with the metric under study are transformed to the form

$$\begin{aligned} \frac{\ddot a}{a} &= -\frac{1}{3}\,(\rho + 3p), \\ H^2 &= \frac{2}{3}\,\rho, \end{aligned} \tag{25}$$

where we have introduced the standard notation for the Hubble constant $H = \dot a/a$ and written the energy-momentum tensor in terms of energy density ρ and pressure p. The covariant conservation of energy-momentum tensor leads to the liquid equation, which follows from (25)

$$\dot\rho a^3 + 3(\rho + p)a^2\dot a = 0. \tag{26}$$

In the given metric we can easily find that[4] $\ln g/\gamma = 6\ln a$.

Let us study the case of dust matter with the cosmological term in RTG. Then

$$\rho_\Lambda = -\frac{3}{4}\,\mu_\omega^2\,\ln a, \quad p_\Lambda = -\rho_\Lambda, \tag{27}$$

$$\rho_M = \frac{3}{4}\,\mu_\omega^2\,\rho_0, \qquad p_M = 0, \tag{28}$$

where we have introduced the dimensionless parameter ρ_0. Note that at $a \equiv 1$ the motion equations are identically satisfied. Equations (25) lead to the expressions

$$\dot H = -\rho_M, \tag{29}$$

$$H^2 = \frac{\mu_\omega^2}{2}\left[\ln\frac{1}{a} + \rho_0\right], \tag{30}$$

and

$$\dot\rho_0 = H\,(1 - 3\rho_0). \tag{31}$$

Putting ρ_0 equal to its stable value[5] at late times we find that the matter density is constant, and

$$\rho_0 = \frac{1}{3}, \quad \rho_M = \frac{\mu_\omega^2}{4}. \tag{32}$$

[4] Since the theory is scale-invariant, for definiteness, we put for the flat metric $a \equiv 1$.

[5] Here it is important that $H > 0$.

Then the time-equations are easily integrated out, so that

$$\begin{aligned} a(t) &= \frac{1}{k} \exp\left[-\left(\sqrt{-\ln(a_0 k)} - \frac{\mu_\omega}{2\sqrt{2}} t\right)^2\right], \\ H^2 &= \frac{\mu_\omega^2}{2}\left(\sqrt{-\ln(a_0 k)} - \frac{\mu_\omega}{2\sqrt{2}} t\right)^2, \\ \dot{H} &= -\frac{\mu_\omega^2}{4}, \end{aligned} \tag{33}$$

where $k = \exp[-\rho_0]$, and the initial data at zero time have to satisfy the condition $\ln(a_0 k) < 0$.

We see that the Universe has got the exponential, inflationary expansion with the constant density of matter, which is significantly less than the density of cosmological term at early times of initial stage. The cosmological term with respect to the flat Minkowskian space-time is naturally interpreted as the contribution of hot strongly coupled gravitons. The density of gravitons falls with the time of inflation, while the entropy is transmitted from the gravitons to the matter, so that the matter entropy grows exponentially.

The deceleration parameter $q = -\frac{\ddot{a}}{a}\frac{1}{H^2}$ is determined by the relation

$$q = -1 - \frac{\dot{H}}{H^2}, \tag{34}$$

and at the initial stage it is close to -1, while in the end of inflation it becomes close to the value observed experimentally. So, for example, we get

$$q\big|_{(\rho_0 - \ln a)=1} = -\frac{1}{2}, \quad q\big|_{(\rho_0 - \ln a)=3/4} = -\frac{1}{3}.$$

The initial conditions of inflation for the flat Universe in RTG are determined at the Planck scale, so that we can naturally suggest that fluctuations are in the following range:

$$\begin{aligned} \delta a &\sim a, \quad a \to 0, \\ \delta t &\sim \frac{1}{m_{\text{PL}}}, \end{aligned}$$

hence,

$$H_0^2 \sim m_{\text{PL}}^2 \implies \ln\frac{1}{a_0} \sim \frac{m_{\text{PL}}^2}{\mu_\omega^2}.$$

This fact implies that at low values of dimensional parameter in RTG the inflation has got a huge scale, that is necessary for an achievement of homogeneous and isotropic thermal equilibrium before the transition to the regime of hot expanding Universe (see [8]).

For the relativistic matter

$$\rho_{RM} = \frac{3}{4}\mu_\omega^2 \rho_{0R}, \qquad p_{RM} = \frac{1}{3}\rho_{RM}, \tag{35}$$

we can write down analogous expressions, such as, for example,

$$\dot{\rho}_{0R} = H\,(1 - 4\rho_{0R}). \tag{36}$$

Choosing the stable value, we get

$$\rho_{0R} = \frac{1}{4}, \quad \dot{H} = -\frac{\mu_\omega^2}{4} = -\frac{4}{3}\,\rho_{RM}. \tag{37}$$

Other expressions for the solution remain valid after the corresponding redefinition of normalization parameter for the relativistic case, so that $k_R = \exp[-\rho_{0R}]$.

Let us stress that the given solutions for the cosmological evolution have been obtained with no reference to the statistical thermal properties of gravitons and matter. For example, let us consider the relations, which take place for the relativistic gas of non-interacting particles. Their densities of energy ρ_M and entropy s_M versus the temperature T are given by the expressions

$$\begin{aligned} \rho_M &= \frac{\pi^2}{30}\mathcal{N}T^4, \\ s_M &= \frac{2\pi^2}{45}\mathcal{N}T^3, \end{aligned} \tag{38}$$

where the number of degrees of freedom is equal to the sum over the polarization states of bosons and fermions with the masses less than the temperature, so that

$$\mathcal{N} = \mathcal{N}_B + \frac{7}{8}\mathcal{N}_F.$$

The presented cosmological solutions show that the ultra-relativistic ideal gas has the constant density of energy, and, hence, it has got a constant temperature. Thus, the inflationary expansion takes place isothermically for the matter, therefore its total entropy increases as $S_M = s_M\, a^3(t)\, V_0$. It is evident that this process can occur only under the loss of entropy by gravitons. The qualitative picture of Universe inflation is the following: in the starting point of inflation at the Planck scale or so, the gravitons have got the large density in comparison with the dilute matter, so that the strong gravitational self-interaction of gravitons leads to a small scattering length (probably about the Planck length), and the gravitons interact with each other, while the matter density is small, and the transparent matter, in practice, evolves isothermically, since the scattering length of gravitons in the matter is much greater than the self-interaction length of gravitons. Therefore, in this approximation we can suppose that the matter, "in practice, weakly" interacts with the gravitational field. In the end of inflation the density o gravitons decreases to the values close to the parameters of matter, i.e. the Universe little deviates from the flat one with the Minkowskian metric. Actually, the graviton scattering length off graviton becomes approximately equal to the scattering length off matter. At this point, the matter state equation differs from that of relativistic particles weakly interacting with the gravity. Probably, the warm gravitons heat up the matter, until the graviton density becomes too low to make a little influence on the cosmological evolution. We see that the Universe inflation results in the dominant interaction of gravitons with the matter. Indeed, the scattering length is defined in terms of cross section σ and particle density n

$$\lambda = \frac{1}{\sigma\, n},$$

therefore, if we naively estimate the particle density by the energy density divided by the temperature

$$n \approx n_0\,\frac{\rho}{T},$$

and put the cross section equal to

$$\sigma = \mathrm{const}\, \frac{T^2}{m_{\mathrm{PL}}^4},$$

then we find the ratio of graviton scattering lengths off the matter and gravitons

$$\frac{\lambda_M}{\lambda_{\mathrm{GR}}} = \frac{T_{\mathrm{GR}}(t)\, \ln \frac{1}{a(t)}}{T_M\, \frac{1}{4}},$$

where the dependence of graviton temperature on the time $T_{\mathrm{GR}}(t)$ is determined by the state equation: density-temperature-volume, for the gravitons. So, we see that at high temperature of gravitons the tending of scale factor a to unity during the inflation leads, initially, to the equalizing of scattering lengths, and further to the dominant interaction of gravitons with the matter.

If the total entropy of gravitational field and matter is conserved, then we have got the following expression for the entropy per a unit volume V_0 (in comoving coordinates independent of time, see the definition of interval):

$$S_0 = a^3(t)\, s_M(T) + S_{\mathrm{GR}}.$$

Further, we write down the standard relation for the total energy of gravitational field

$$d\mathcal{E}_{\mathrm{GR}} = T_{\mathrm{GR}}\, dS_{\mathrm{GR}} - p_\Lambda\, dV, \tag{39}$$

where, evidently,

$$\begin{aligned} V(t) &= a^3(t), \\ \mathcal{E}_{\mathrm{GR}} &= \rho_\Lambda(t)\, a^3(t), \\ p_\Lambda &= -\rho_\Lambda, \end{aligned} \tag{40}$$

so that after taking the derivative with respect to time we find

$$T_{\mathrm{GR}} = -\frac{V(t)\, \dot\rho_\Lambda}{s_M(T)\, \dot V(t)} = \frac{\mu_\omega^2\, m_{\mathrm{PL}}^2}{4 s_M(T)}.$$

Therefore, we have got that the temperature of gravitons also does not change during the time of inflation

$$T_{\mathrm{GR}} = \frac{45}{8\pi^2 N(T)}\, \frac{\mu_\omega^2\, m_{\mathrm{PL}}^2}{T^3},$$

and the process is isothermic for the gravitons, too. Moreover, comparing the state equation of matter (38) with the stable density of matter in the inflation

$$\rho_M = \frac{3}{16}\, \mu_\omega^2\, m_{\mathrm{PL}}^2,$$

we find that

$$T_{\mathrm{GR}} = T,$$

i.e. the temperature of gravitons is equal to the temperature of matter during the inflation.

The equality of temperatures and the adiabaticity of process can be obtained also from the general equation of state (39) combined for the gravitons and matter, if we take into account

(40) and analogous equations for the relativistic matter with the stable density. Indeed, calculating the differentials in the equation

$$d\mathcal{E}_{\rm GR} + d\mathcal{E}_M = (T_{\rm GR} - T_M)\, dS_{\rm GR} - p_\Lambda\, dV - p_M\, dV,$$

we get

$$(T_{\rm GR} - T_M)\, dS_{\rm GR} = 0,$$

so that, if the process is adiabatic for the closed system, then the temperatures of gravitons and matter are equal to each other.

Further, we have to emphasize two circumstances. First, at $H < 0$, i.e. after the end of inflationary isothermic expansion and the transition to the contraction, the point of constant density of matter $\rho_{0\rm R}$ becomes **unstable** in accordance with (36). Second, for the gauge non-gravitational interactions of matter with the coupling constant α we can estimate the cross section by

$$\sigma_{\rm gauge} = \sigma_0\, \frac{\alpha}{T^2},$$

so that the scattering length in such interactions of matter is significantly less than the graviton scattering length off matter,

$$\lambda_{\rm gauge} = \frac{1}{\sigma_0\, \alpha\, T} \ll \lambda_M \sim \frac{m_{\rm PL}^4}{T^5}, \tag{41}$$

This fact implies that in the beginning of inflation, when the graviton scattering off gravitons dominates, the isothermic adiabatic expansion takes place, while in the end of inflation, when the graviton scattering length off matter closes to the self-scattering length, the gravitons begin to lose the energy and heat up the matter, so that the absorbed energy of gravitons is transformed into the thermal motion due to the gauge interactions because of (41). Therefore, both the density and entropy of matter grow in the comoving coordinates. This increase is permitted, since the stability of constant value for the matter density is destroyed. By the process of such heating, at the time t_1 the energy density takes the form

$$\rho_{\rm I} = \frac{3}{4}\, \tilde{\mu}_{\rm I}^2\, m_{\rm PL}^2 \left(\ln \frac{1}{a_1\, k_{\rm R}} + \frac{\Delta\rho_{\rm IR}}{a_1^4}\right), \tag{42}$$

$$\rho_{\rm II} = \frac{3}{4}\, \tilde{\mu}_{\rm II}^2\, m_{\rm PL}^2 \left(\ln a_1\, k_{\rm R} + \frac{\Delta\rho_{\rm IIR}}{a_1^4}\right), \tag{43}$$

where $a_1 = a(t_1)$, and by (36) we put

$$\rho_{\rm IR}(t) = \frac{1}{4} + \frac{\Delta\rho_{\rm IR}}{a^4(t)}, \quad \rho_{\rm IM} = \frac{3}{4}\, \tilde{\mu}_{\rm I}^2\, m_{\rm PL}^2 \left(\frac{1}{4} + \frac{\Delta\rho_{\rm IR}}{a^4}\right), \tag{44}$$

$$\rho_{\rm IIR}(t) = \frac{1}{4} - \frac{\Delta\rho_{\rm IIR}}{a^4(t)}, \quad \rho_{\rm IIM} = \frac{3}{4}\, \tilde{\mu}_{\rm II}^2\, m_{\rm PL}^2 \left(-\frac{1}{4} + \frac{\Delta\rho_{\rm IIR}}{a^4}\right), \tag{45}$$

where $\Delta\rho_{\rm R}$ is the constant of integration, which we define in accordance with two variants of process development, since the parameter $\tilde{\mu}^2$ modified due to the graviton absorption can take two signs: in the second variant this parameter changes the sign in comparison with its state before the absorption μ_ω^2, while in the first variant the sign remains with no change.

The physical sense of these two variants will be shown below under the description of their cosmological differences.

The nature of μ^2 fall off may be twofold. First, we stress the essential fact of instability for the matter density, so that the absorption of gravitons can play a role of stabilization, and under the change of sign $\mu_\omega^2 \to -\tilde{\mu}_{\rm II}^2$ the stability of potential is restored. Second, it may be important that the changing scale of evolution $a(t)$ corresponds to an introduction of renormalization group scale dependence of matter charges and Yukawa-like couplings, so that the physical phase state rearrangement is possible, since such critical parameters as the temperature of phase transition, in general, depend on the mentioned running coupling constants.

A powerful absorption of gravitons leads to

$$\left|\ln\frac{1}{a_1\, k_{0\rm R}}\right| \ll \frac{\Delta\rho_{\rm R}}{a_1^4},$$

and we get the evolution equation

$$H^2 = \frac{1}{2}\,\tilde{\mu}_{\rm I,II}^2\, m_{\rm PL}^2\,\frac{\Delta\rho_{\rm I,IIR}}{a^4(t)}, \tag{46}$$

that implies the transition to the scenario of hot expanding Universe, because

$$a^2(t) = a_1^2 + \sqrt{2}\,\tilde{\mu}_{\rm I,II}\, m_{\rm PL}\,(t - t_1), \tag{47}$$

and at late times for the relativistic matter we obtain

$$a(t) \sim \sqrt{t}.$$

The temperature of hot Universe T_{hot} is determined from the comparison of energy density in the thermal equilibrium with the expression given in terms of $\rho_{0\rm R}$, so that

$$\frac{\pi^2}{30}\,\mathcal{N}\, T_{hot}^4 \approx \frac{3}{4}\,\tilde{\mu}_{\rm I,II}^2\, m_{\rm PL}^2\,\frac{\Delta\rho_{\rm R}}{a_1^4},$$

wherefrom we have got an ordinary relation

$$T(t) \sim \frac{1}{a(t)}.$$

Since the absorption of gravitons by the matter takes place at the constant volume and with no loss of energy, we have introduce the notation $\tilde{\mu}$ for the dimensional parameter after the absorption, so that, evidently, comparing the total energy before the absorption of gravitons and after it, we get

$$\tilde{\mu}_{\rm I}^2 \approx \tilde{\mu}_{\rm II}^2 \approx \mu_\omega^2\,\frac{\ln\frac{1}{a_1\, k_{\rm R}}}{\frac{\Delta\rho_{\rm R}}{a_1^4}} \quad\Rightarrow\quad \tilde{\mu}^2 \ll \tilde{\mu}_\omega^2.$$

Thus, the stage of hot expanding Universe can be quite long in time in order to have no contradiction with the astronomical observations.

The hierarchy of scales μ_ω and $\tilde{\mu}$ implies also that at the graviton absorption

$$\mu_\omega^2\,\frac{1}{4} \approx \tilde{\mu}^2\,\frac{\Delta\rho_{\rm R}}{a_1^4},$$

and the heating up the matter is little in comparison with the stage of inflation.

Further, the given consideration can be analogously performed for the dust matter having a zero pressure. Then we get

$$\rho_0(t) = \frac{1}{3} \pm \frac{\Delta\rho_0}{a^3(t)},$$

and the law of hot Universe expansion is ordinary again

$$a(t) \sim \sqrt[3]{t^2}.$$

Let us also consider the problem on the reaching the "critical" zero density of matter in the second scenario of postinflationary evolution. Indeed, at $a(t) > \sqrt[4]{4\Delta\rho_{\rm IIR}}$ the density of matter formally becomes negative, that has no physical sense for nonzero modes, if we do not suppose that this situation is possible under, first, the vacuum energy is negative in the state equations of matter, and, hence, the null-point of energy measure is posed at a negative energy; second, it is necessary that $\ln a \gg 1$ and a correct cosmological solution could take place: $H^2 = \frac{2}{3}\rho > 0$, though, as clear from the astronomical observations, this condition is valid.

The notion on the connection of matter density with the nonzero value of effective potential in the state of matter vacuum is important, because we see that at the inflation the stable isothermic density of matter energy $V_M^{inf}(0) \sim \mu_\omega^2 m_{\rm PL}^2 > 0$, while in the hot Universe the matter density tends to a stable value of $V_{\rm IM}(0) > 0$ or $V_{\rm IIM}(0) < 0$ depending on the variant of development.

Finally, we note also that the one-loop quantum corrections lead to a renormalization of cosmological constant [9], so that in RTG they determine the adiabatic dependence of dimensional parameter μ_ω on the scale a, since it can be related with the ultraviolet cut off M by the formula $d\ln M = -d\ln a$. In this work we do not discuss a motivation and numerical estimates of dimensional parameters for the cosmological evolution in RTG.

Thus, we have analyzed the cosmological solutions in the modified RTG and found the conditions of inflationary expansion as well as transition to the ordinary evolution of hot Universe.

4 Conclusion

In the present paper we have shown that the cosmological scenario of Universe evolution possesses some advantages in the relativistic theory of gravitation, which naturally introduces the dependence of gravitational field action on the vacuum metric of Minkowski at nonzero cosmological term necessary in accordance with the astronomical observations. Certainly, the evolution equations allow the development of logically sound stages providing the inflationary expansion and the modern epoch of late hot Universe. In this way the inflation takes place isothermically, and we have not to introduce a mechanism for a forced heating up a overcooled homogeneous and isotropic Universe under the transition from the inflation to the hot Universe. In RTG the following stages of cosmological evolution are theoretically sound:

1. The beginning: a hot state with a Planck-scale density of both ultra-relativistic matter and gravitons; $\rho_M \approx \rho_\Lambda \sim m_{\rm PL}^4$ with a temperature of $T_{\star\rm GR} \sim T_{\star M} \sim m_{\rm PL} \sim 10^{19}$ GeV in a strongly compressed state $\ln\frac{1}{a_\star} \gg 1$; a time $t_\star$.

2. The preinflation: a stage of expansion and matter cooling down a temperature T_M, so that $T_M^4 \sim \mu_\omega^2 m_{\text{PL}}^2$, where $\mu_\omega^2 \sim \frac{m_{\text{PL}}^2}{-\ln a_\star} \ll m_{\text{PL}}^2$; a scale of $T_M \sim 10^{12}$ GeV; the time changes from $t_\star$ to t_0, whereas $\ln \frac{1}{a(t_0)} \gg 1$.

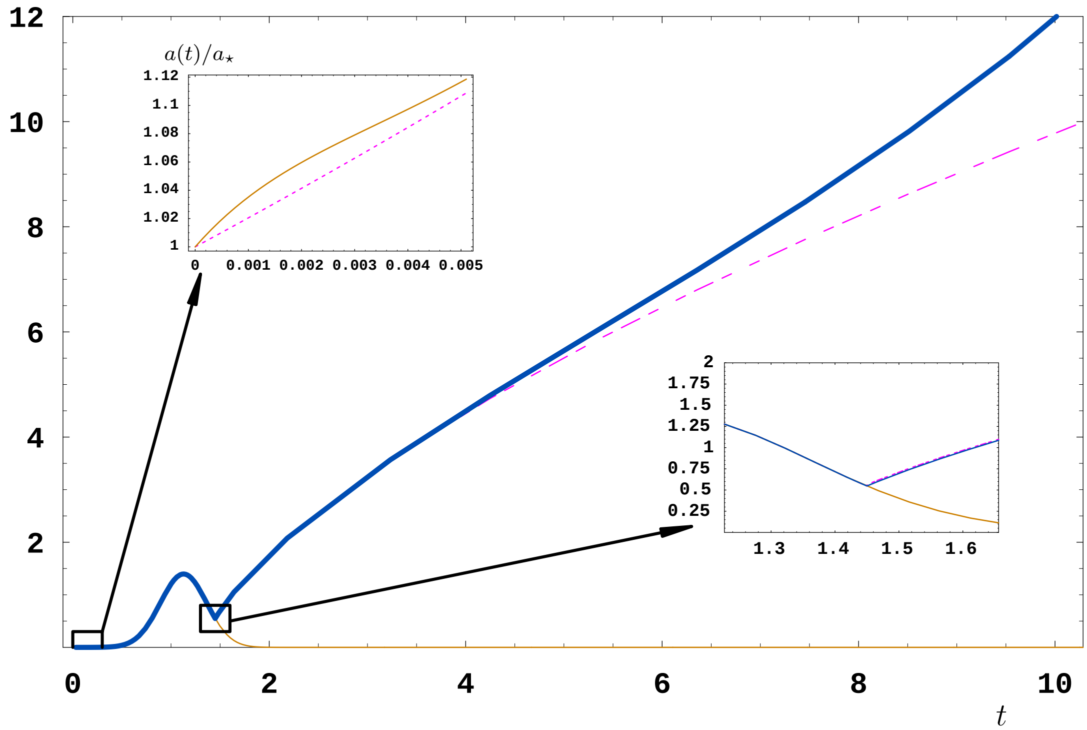

Figure 3: Stages of Universe evolution in accordance with items 1-6. In the begin stage we show the scenario of cooling down below the Planck temperatures before the inflation in comparison with the pure inflation (the dotted line). The unstable regime change takes place under the graviton absorption to the process of hot Universe expansion in comparison with the rapid exponential contraction. At late times we present the deviation from the regime of hot Universe expansion (the dashed curve) due to a cold inflationary expansion (the solid curve). The time t and scale $a(t)$ units are modelled and far away from the real ones.

3. The isothermic hot inflation with $T_{\text{GR}} = T_M$ up to a time t_1, when the density of gravitons falls off a level of matter density $|\ln a(t_1)| \sim 1$.

4. The short stage of instability $H < 0$, when the energy of gravitons is converted to the thermal energy of matter due to the absorption of gravitons, so that before the absorption $\rho_{\text{GR}}(t_1) \sim \rho_M(t_1)$, while after it $\tilde{\rho}_{\text{GR}} \ll \tilde{\rho}_M$; the restoration of stability[6] $H > 0$, whereas the matter is slightly heated up, and the evolution parameter $\tilde{\mu}^2/\mu_\omega^2 \sim \tilde{\rho}_{\text{GR}}/\rho_{\text{GR}} \ll 1$.

[6]For the relativistic matter the evolution equations:

$$H^2 = \frac{2}{3}(\rho_M + \rho_{\text{GR}}), \quad \frac{\ddot{a}}{a} = H^2 - \frac{4}{3}\rho_M,$$

result in the inflection point at $\rho_{\text{GR}} = \rho_M$, while after the graviton absorption we get $\tilde{\rho}_M \gg \tilde{\rho}_{\text{GR}}$, so that the transition from the concave regime of contraction to the convex curve of expansion is possible (see Fig. 3).

5. The present: the epoch of hot Universe expansion with the matter temperature $T(t) \sim \frac{1}{a(t)}$ down to a temperature T_c, so that $T_c^4 \sim \tilde{\mu}^2 m_{\rm PL}^2$. Today we are "close" to the end of this stage because in the case of nonrelativistic cosmological motion of matter the deceleration parameter

$$q = -1 + \frac{3}{2}\frac{\rho_M}{\rho_M + \rho_{\rm GR}} \approx -0.33 \pm 0.17,$$

gives the witness that $\rho_{\rm GR} \sim (1-2)\rho_M$, i.e. the matter density is critically low.

6. The future II: a stable density of matter energy $V_M(0) < 0$, the infinite cold isothermic inflation with an logarithmically small density of matter. The speed of inflation here could be "demped" by a parametric adiabatic decrease of parameter $\tilde{\mu}^2$ down to zero because of a renormalization scale-dependent behaviour, that suggests the tending of matter vacuum density to zero, $V_M(0) \to 0$. This scenario of future with $V_M(0) < 0$ is the most probable in the light of astronomical data on the deceleration parameter, which is negative at the moment.

The future I: a stable density of matter energy $V_M(0) > 0$, the expansion of Universe up to the equalizing of matter density with the negative contribution of cosmological term, which provides $q > \frac{3}{2}$, that contradicts the current data, the later stage of instability of cold Universe $H < 0$, so that whether the restoration of stability will lead to an expansion again as in items 4-5, but for the overcooled Universe with a further repetition of transition to a future according to item 6, or the contraction of Universe will take place with significant fluctuations of density for the matter heated up.

Thus, in RTG we have got quite the whole picture of Universe evolution.

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